Mixed gauge and anomaly mediation from new physics at 10 TeV

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ABSTRACT: In the context of anomaly-mediated supersymmetry breaking, it is natural for vectorlike fields and singlets to have supersymmetry breaking masses of order 10 TeV, and therefore act as messengers of supersymmetry breaking. We show that this can give rise to phenomenologically viable spectra compatible with perturbative gauge coupling unification. The minimal model interpolates continuously between pure anomaly mediation and gauge mediation with a messenger scale of order 10 TeV. It is also possible to have non-minimal models with more degenerate specta, with some squarks lighter than sleptons. These models reduce to the MSSM at low energies and incorporate a natural solution of the μ problem. The minimal model has four continuous parameters and one discrete parameter (the number of messengers). The LEP Higgs mass bound can be satisfied in the minimal model by tuning parameters at the GUT scale to one part in 50.

KEYWORDS: Supersymmetry Breaking, Supersymmetry Phenomenology.

Contents

1. Introduction

Anomaly-mediated supersymmetry breaking (AMSB) [1, 2] is an attractive mechanism for breaking supersymmetry (SUSY) without flavor problems. In this mechanism, SUSY is broken by the VEV of a supergravity auxiliary field $\langle F_\phi \rangle$. The supercovariance rules that govern the couplings of this auxiliary field are easily summarized: $\phi = 1 + \theta^2 F_\phi$ is a superconformal compensator, a chiral superfield with conformal dimension 0 and conformal $\mathrm{U}(1)_R$ charge $+\frac{2}{3}$ (in a normalization where the superpotential has R charge $+2$). Matter and gauge chiral superfields have conformal dimension equal to their mass dimension, and vanishing conformal R charge. From this it is clear that the coupling of the conformal compensator is independent of flavor, and therefore anomaly mediation predicts a flavor-blind SUSY breaking spectrum. The SUSY breaking masses depend on scale via the renormalization group (RG), and anomaly mediation defines a preferred trajectory for all SUSY breaking couplings in terms of a single SUSY breaking scale $\langle F_\phi \rangle \sim 10$ TeV. Unfortunately, the slepton mass parameters are negative in the minimal supersymmetric standard model (MSSM). In this paper, we propose a solution to this problem based on an idea due to Nelson and Weiner [3], which built on early work by Pomarol and Rattazzi [4]. Nelson and Weiner considered a theory with extra vectorlike fields P and $\tilde{P}$ and added a coupling of the form[1]

$$\Delta\mathcal{L} = \int d^4\theta \, \frac{\phi}{\phi^\dagger} c P \tilde{P} + \text{h.c.} \tag{1.1}$$

$$= c\langle F_\phi \rangle^\dagger \int d^2\theta \, P\tilde{P} + c|\langle F_\phi \rangle|^2 P\tilde{P} + \text{h.c.} \tag{1.2}$$

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[1]Couplings of this form with P and $\tilde{P}$ replaced by the MSSM Higgs fields contribute to the Giudice-Masiero mechanism for generating the MSSM μ term [5].

We see that this gives a SUSY invariant superpotential mass term for P and $\tilde{P}$, as well as a SUSY-violating 'B-type' scalar mass term. Diagonalizing the scalar mass matrix, we find that the scalar mass-squared eigenvalues are positive for $|c| > 1$. Assuming $|c| \sim 1$, this is a supersymmetry breaking threshold at the scale $\langle F_\phi \rangle$, which gives SUSY breaking threshold corrections of order $g^2 \langle F_\phi \rangle / 16\pi^2$ to SUSY breaking masses, taking them off the AMSB RG trajectory. As shown in ref. [3], the leading threshold corrections to the scalar masses vanish, and the slepton mass-squared terms are therefore still negative at the scale $\langle F_\phi \rangle$. One can get positive slepton masses at the weak scale only by having a large number of messengers (5 or more $\mathbf{5} \oplus \mathbf{\bar{5}}$'s), which generates large gaugino masses at the messenger scale ~ 10 TeV, which in turn generates positive slepton masses from running between the messenger scale and the weak scale. However, the resulting theories generally have charged slepton LSP, and the large number of messengers destroys perturbative unification.

In this paper we consider a very simple extension of this model that has a more attractive phenomenology. The model consists of the MSSM plus a singlet S in addition to the vectorlike fields P, $\tilde{P}$. We include the most general interactions with dimensionless coefficients. The additional terms in the Lagrangian are therefore

$$\Delta\mathcal{L} = \int d^4\theta \frac{\phi^\dagger}{\phi} \left(\tfrac{1}{2} c_S S^2 + c_P P \tilde{P} \right) + \text{h.c.}$$
$$+ \int d^2\theta \left[\frac{\lambda_S}{3!} S^3 + \lambda_P S P \tilde{P} \right] + \text{h.c.}$$
(1.3)

A superpotential coupling of the form $S H_u H_d$ is assumed to be absent.[2] For $|c_S| < 1$ the potential for S has a local maximum at $S = 0$, so $\langle S \rangle \neq 0$. This gives rise to a more general threshold with none of the problems of the minimal model.

2. The threshold

In this section, we compute the SUSY breaking from the threshold. The scalar potential that arises from eq. (1.3) is

$$V = \left| c_S \langle F_\phi^\dagger \rangle S + \tfrac{1}{2} \lambda_S S^2 + \lambda_P P \tilde{P} \right|^2 + |c_P \langle F_\phi^\dagger \rangle + \lambda_P S|^2 (|P|^2 + |\tilde{P}|^2)$$
$$+ |\langle F_\phi \rangle|^2 \left(\tfrac{1}{2} c_S S^2 + c_P P \tilde{P} \right) + \text{h.c.}$$
(2.1)

The potential is quadratic in P, $\tilde{P}$, so we look for a minimum with $\langle P \rangle = \langle \tilde{P} \rangle = 0$. In the appendix, we minimize the potential for real couplings and VEVs. We show that the global minimum preserves CP for

$$c_S < 0 \tag{2.2}$$

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[2]For example, it may be forbidden by a discrete R symmetry $S(\theta) \mapsto -S(i\theta)$, $P(\theta) \mapsto +P(i\theta)$, $\tilde{P}(\theta) \mapsto +\tilde{P}(i\theta)$, $H_u(i\theta) \mapsto +H_u(i\theta)$, $H_d(i\theta) \mapsto -H_d(i\theta)$, $u^c(\theta) \mapsto -u^c(i\theta)$, with all other fields even.

and the we obtain

$$\langle S \rangle = -\frac{\langle F_\phi \rangle}{2\lambda_S} \left(3c_S + \sqrt{c_S(c_S - 8)} \right), \tag{2.3}$$

$$\left\langle \frac{F_S}{S} \right\rangle = \frac{\langle F_\phi \rangle}{4} \left(-c_S + \sqrt{c_S(c_S - 8)} \right). \tag{2.4}$$

This gives rise to a mass term for P, $\tilde{P}$ that can be conveniently written as

$$\Delta\mathcal{L} = \int d^2\theta \, \phi \mathcal{M} P \tilde{P} + \text{h.c.}, \tag{2.5}$$

where

$$\mathcal{M} = M[1 + \theta^2 r \langle F_\phi \rangle], \tag{2.6}$$

In this parameterization $r \neq 0$ parameterizes the deviation from a supersymmetric threshold, i.e. $r = 0$ gives a pure anomaly-mediated spectrum below the messenger scale. The model of Nelson and Weiner has $r = -2$. We then have

$$M = c_P(1 + X)\langle F_\phi \rangle, \tag{2.7}$$

$$r = -\frac{2 + \frac{1}{4}X \left(c_S + 4 - \sqrt{c_S(c_S - 8)} \right)}{1 + X}, \tag{2.8}$$

where

$$X = \frac{\lambda_P \langle S \rangle}{c_S \langle F_\phi \rangle} = -\frac{\lambda_P}{2c_P \lambda_S} \left(3c_S + \sqrt{c_S(c_S - 8)} \right). \tag{2.9}$$

This shows that all values of M and r are allowed, since $1 + X$ can be small and have either sign. (Note that this does not require any Yukawa couplings to be large.) In order to avoid a negative mass eigenvalue for the scalars P, $\tilde{P}$ at the minimum, we require

$$|(r + 1)\langle F_\phi \rangle| < |M|. \tag{2.10}$$

We now evaluate the threshold contributions to the standard model fields due to the P fields. The general formulas can be obtained from the methods of refs. [6, 7]. The soft SUSY breaking terms can be parameterized by higher superspace components of dimensionless couplings via

$$m_0^2 = -\frac{\partial}{\partial\theta^2} \frac{\partial}{\partial\bar{\theta}^2} \ln Z, \tag{2.11}$$

$$m_{1/2} = \frac{1}{g} \frac{\partial}{\partial\theta^2} g, \tag{2.12}$$

$$\lambda A = -2 \frac{\partial}{\partial\theta^2} \lambda, \tag{2.13}$$

where all couplings are taken to be real superfields. In the present model, all SUSY breaking is contained in the conformal compensator and the P, $\tilde{P}$ mass term, so we have

$$\frac{\partial}{\partial\theta^2} = \frac{1}{2}\langle F_\phi \rangle \left(r \frac{\partial}{\partial \ln M} - \frac{\partial}{\partial \ln \mu} \right). \tag{2.14}$$

Note that this implies the presence of mixed anomaly- and gauge-mediated terms for scalar masses, as first pointed out in ref. [4]. In this way, we can obtain expressions for the soft masses at the scale M in the effective theory where P and $\tilde{P}$ have been integrated out:

$$m_0^2(M) = \frac{1}{4}\langle F_\phi \rangle^2 \left\{ -r^2 \frac{\partial \gamma'}{\partial g_i'} \beta_i' + 2r(r+1)\frac{\partial \gamma}{\partial g_i}\beta_i' - (r+1)^2 \frac{\partial \gamma}{\partial g_i}\beta_i \right\}, \qquad (2.15)$$

$$m_{1/2}(M) = \frac{1}{g}\langle F_\phi \rangle \left[r\beta_g' - (r+1)\beta_g \right], \qquad (2.16)$$

$$A(M) = -\frac{1}{\lambda}\langle F_\phi \rangle \left[r\beta_\lambda' - (r+1)\beta_\lambda \right]. \qquad (2.17)$$

Here primed (unprimed) quantities refer to the theory above (below) the scale M. The anomalous dimensions are defined by

$$\beta_i = \frac{\partial g_i}{\partial \ln \mu}, \qquad \gamma = \frac{\partial \ln Z}{\partial \ln \mu}. \qquad (2.18)$$

The expression for the scalar masses can be simplified in the case of fields with no Yukawa couplings to messengers, for which $\gamma' = \gamma$. We then have

$$m_0^2(M) = m_{0\,\text{AMSB}}^2 + \frac{1}{4}r(r+2)\langle F_\phi \rangle^2 \frac{\partial \gamma}{\partial g_i}\Delta\beta_i, \qquad (2.19)$$

where

$$m_{0\,\text{AMSB}}^2 = -\frac{1}{4}\langle F_\phi \rangle^2 \frac{\partial \gamma}{\partial g_i}\beta_i. \qquad (2.20)$$

and $\Delta\beta = \beta' - \beta$. Similarly, we can write

$$m_{1/2}(M) = m_{1/2\,\text{AMSB}} + \frac{r}{g}\langle F_\phi \rangle \Delta\beta_g \qquad (2.21)$$

$$A(M) = A_{\text{AMSB}} - \frac{r}{\lambda}\langle F_\phi \rangle \Delta\beta_\lambda. \qquad (2.22)$$

These expressions explicitly display the fact that the soft masses reduce to the AMSB values in the limit $r \to 0$. The scalar masses (but not gaugino masses and A terms) also reduce to their AMSB values for $r \to -2$, as in the model of Nelson and Weiner. In the generalized model, all soft masses reduce to the gauge-mediated values in the limit $r \to \infty$ with $r\langle F_\phi \rangle$ held fixed. For general r, the SUSY breaking spectrum in this model interpolates continuously between anomaly mediation and gauge mediation with a messenger scale of order 10 TeV (assuming all dimensionless couplings are order unity).

As with the case of pure gauge- and anomaly-mediated SUSY breaking, eqs. (2.15)–(2.17) are leading order results in a power series with subleading corrections suppressed by $\mathcal{O}((\langle F_\phi \rangle/M^2)^2)$ and $\mathcal{O}((r\langle F_\phi \rangle/M^2)^2)$. In the present class of models, it is natural to have $M \sim \langle F_\phi \rangle, r\langle F_\phi \rangle$, where these effects may be important. They have been calculated for the case of pure gauge mediation, where they are known to be numerically small unless the SUSY breaking is tuned to be close to the instability limit $F/M^2 \to 1$ [8]. Because these corrections are UV finite, they do not depend on the regulator, and therefore depend

on the conformal compensator only through the superfield mass of the messengers (see eq. (2.5)). We can therefore use the results for gauge mediation with the replacement $F/M^2 \rightarrow (r+1)\langle F_\phi \rangle/M^2$. Since the stability limit is $|(r+1)\langle F_\phi \rangle/M^2| < 1$ here as well, the corrections are small in the absence of fine tuning.

3. The μ problem

In the context of AMSB, we cannot get a phenomenologically acceptable Higgsino mass by adding a μ term

$$\Delta \mathcal{L} = \int d^2\theta \, \mu\phi H_u H_d + \text{h.c.} \tag{3.1}$$

since this gives rise to $B \sim \langle F_\phi \rangle \sim 10$ TeV. One possibility is the NMSSM, where the VEV of a singlet gives the μ term. However, it is nontrivial to get a negative mass-squared term for the singlet. Here we briefly discuss another possibility within the MSSM that gives a more minimal model.

We consider a mechanism originally proposed by Randall and Sundrum in ref. [1]. We show that this mechanism can be made natural with appropriate broken symmetries. We add a term to the Lagrangian of the form

$$\Delta \mathcal{L}_{\text{RS}} = \int d^4\theta \, c(Y + Y^\dagger)\frac{\phi^\dagger}{\phi} H_u H_d + \text{h.c.} \tag{3.2}$$

Here we have included factors of ϕ by canonically normalizing $H_{u,d}$ but not the field Y. Expanding this out, we obtain the potential terms

$$\Delta \mathcal{L}_{\text{RS}} = \left[-c|F_\phi|^2(Y + Y^\dagger) + c(F_\phi^\dagger F_Y - \text{h.c.}) \right] H_u H_d + \text{h.c.}$$
$$+ \left[cF_\phi^\dagger(Y + Y^\dagger) + cF_Y^\dagger \right] \int d^2\theta \, H_u H_d + \text{h.c.} \tag{3.3}$$

We see that we can naturally get a vanishing $B\mu$ term at tree level if

$$\langle Y + Y^\dagger \rangle = 0 \tag{3.4}$$

and all couplings and VEV's are real. This is natural by CP invariance, and we then obtain an effective μ term

$$\mu = c\langle F_Y \rangle. \tag{3.5}$$

The $B\mu$ term is generated from AMSB, giving rise to a model with only one additional parameter.

It is crucial that the Y appears in the combination $Y + Y^\dagger$. This is natural if the field Y is invariant under a shift symmetry

$$Y \mapsto Y + i\lambda \tag{3.6}$$

where λ is a real constant. We must also forbid a term of the form

$$\Delta\mathcal{L} = \int d^4\theta \, c' \frac{\phi^\dagger}{\phi} H_u H_d + \text{h.c.} \tag{3.7}$$

The discrete R symmetry

$$Y(\theta) \mapsto -Y(i\theta), \quad H_u(\theta) \mapsto H_u(i\theta), \quad H_d(\theta) \mapsto -H_d(i\theta) \tag{3.8}$$

forbids the unwanted term eq. (3.7), and also has the feature that the lowest component of Y is odd, while F_Y is even.[3] The VEV for Y that we need is therefore protected by this symmetry. In order to make the Yukawa couplings invariant, the standard model fields must also transform under the discrete symmetry, e.g.

$$u^{\text{c}}(\theta) \mapsto -u^{\text{c}}(i\theta), \tag{3.9}$$

with all other fields even.

This shows that the term eq. (3.2) with Y treated as a spurion provides a viable μ term in AMSB that is natural by symmetries. Effectively, it justifies the inclusion of a running μ term into the AMSB RG trajectory. It does not explain why the μ term is the same size as other SUSY breaking terms. We leave this for future work.

4. Spectrum and phenomenology

We now discuss the SUSY breaking spectrum that results from this model. We assume that the messengers come in complete SU(5) multiplets, so that the gauge coupling unification in the MSSM is not an accident. The simplest possibility is then that the messengers consist of N copies of $\mathbf{5} \oplus \bar{\mathbf{5}}$. For perturbative unification, we require $N \leq 4$. Under the standard model gauge group, these decompose into a doublet and a triplet, each of which can have different couplings c_P and λ_P (see eq. (1.3)). These give rise to different values for r for the doublet and triplet messengers, and hence different SUSY breaking masses for colored and uncolored superpartners. We assume for simplicity that the N messengers have the same coupling (e.g. there can be an unbroken SU(N) symmetry in the messenger sector). This can be relaxed to obtain even more general spectra.

For large r, the spectrum is close to that of gauge mediation. However, because SUSY breaking is driven by anomaly mediation, the gravitino mass is naturally of order $\langle F_\phi \rangle$, alleviating the gravitino problem. This may not be large enough for large r, but it is possible (and natural) to have masses for the gravitino and other gravitational moduli that are parametrically larger than $\langle F_\phi \rangle$ with SUSY breaking dominated by anomaly mediation [9].

The simplest model is completely specified at high energies by M, F_ϕ, r_2, r_3, N, μ, and y_t. There are two low-energy constraints from the known values of the Higgs VEV $v^2 = v_u^2 + v_d^2$ and the top quark mass, so this model has four continuous parameters and one discrete parameter. Of these, the dependence on the messenger scale M is only logarithmic,

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[3]This symmetry also forbids unwanted couplings between the singlet S and the Higgs fields if S transforms as $S(\theta) \mapsto -S(i\theta)$.

since it just sets the scale for the RG running down to the weak scale. Explicit formulas for soft masses are presented in appendix B.

For illustration, the spectrum of superpartner masses at the messenger scale is shown in figure 1 as a function of $r = r_2 = r_3$ for $M = 50\,\text{TeV}$, for $N = 1$ and $N = 4$ respectively. For $r < 0$ we can obtain positive slepton mass-squared parameters, but the right-handed sleptons are lighter than the bino, giving rise to charged slepton LSP. We therefore focus our attention on $r > 0$. The spectra are still qualitatively similar to gauge- and anomaly mediation in the sense that colored superpartners are heavier than uncolored ones. For example, obtaining positive slepton mass-squared parameters requires $r \gtrsim 1$, which then implies $m_{\tilde{q}} \gtrsim 5m_{\tilde{\ell}}$.

Quite different possibilities exist if $r_2 \neq r_3$. In figure 2 we show an example spectrum with $N = 1$ and $r_3 = -1$. We again require $r > 0$ to avoid a slepton LSP. We see that the spectrum is more degenerate, and the $\mathrm{SU}(2)_W$ contribution to superpartner masses is comparable to $\mathrm{SU}(3)_C$. For $r_2 \gtrsim 2$, the superpartners charged under $\mathrm{SU}(2)_W$ are the heaviest, followed by the gluino, then right-handed scalars and the Bino. Such spectra open up new regions of SUSY parameter space that may be interesting to explore. These spectra have a light stop, and therefore requires an additional contribution to the Higgs quartic. Possibilities include a "fat" Higgs [10] or large D terms from exotic gauge interactions [11].

We give some representative points in parameter space in table 1, assuming $r_2 = r_3$ for simplicity. At the scale M we evaluate the soft-breaking parameters using eqs. (2.15)–(2.17), and evolve them down using MSSM RG equations to the stop mass scale $m_{\tilde{t}}$. (Since we have small mixing in the stop sector, we simply use the common stop mass.) At the scale $m_{\tilde{t}}$, we determine the μ parameter by minimizing the one-loop effective potential. This includes the largest 2-loop corrections to the effective potential because we use a value of y_t that includes 1-loop QCD corrections [12]. We then add by hand the 2-loop QCD threshold corrections to the higgs mass $m_{h^0}^2$, although this is a small correction (< 2 GeV) for small stop mixing.

The spectra given in table 1 satisfy all experimental constraints. The most severe constraint is the LEP Higgs mass bound $m_{h^0} > 114.4\,\text{GeV}$. Because we do not have large mixing in the stop-sector, we require $m_{\tilde{t}} \sim 1\,\text{TeV}$ to satisfy the Higgs mass bound, and the experimental constraints on the sleptons and LSP are easily satisfied. As we have large stop masses, these models are fine-tuned.

We quantify the fine tuning by the sensitivity of the Higgs to varying parameters at the GUT scale. The Higgs mass is quadratically sensitive to the stop mass, but this is not a fundamental parameter in this model. The most sensitive fundamental parameter is $g_3(M_{\text{GUT}})$, so we define

$$\text{Fine tuning} \equiv \frac{g_3(M_{\text{GUT}})}{v}\frac{\partial v}{\partial g_3(M_{\text{GUT}})} = \frac{\partial \ln v}{\partial \ln g_3(M_{\text{GUT}})}. \tag{4.1}$$

Because the sensitivity is through the stop mass, the tuning increases quadratically with the stop mass, while the lightest Higgs mass increases only logarithmically. This means that the fine tuning increases exponentially as a function of the lightest Higgs mass. This
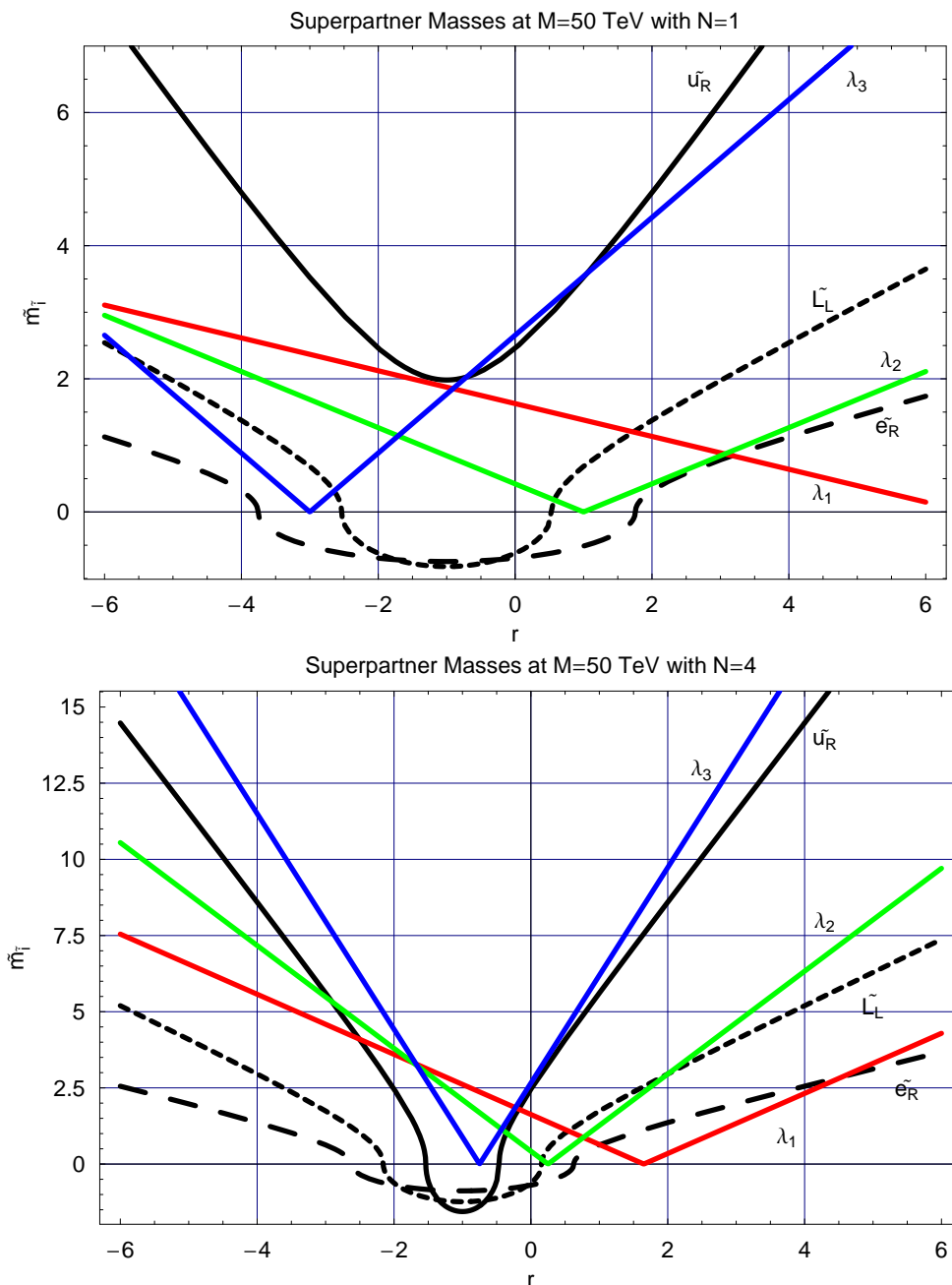
Figure 1: Spectrum of superpartner masses as a function of $r = r_2 = r_3$ for $M = 50$ TeV, and $N = 1$ (top) and $N = 4$ (bottom). For gaugino masses we plot $|M|$ and for scalar masses, we plot $|m^2|^{1/2} \times \mathrm{sgn}(m^2)$. All masses are in units of $F_\phi/(16\pi^2)$.

phenomenon is intrinsic to the MSSM, not just the present model, and is illustrated in figure 3. Note that the fine-tuning is somewhat less for a large number of messengers, since QCD is non asymptotically free in this case, and therefore the sensitivity to $g_3(M_{\mathrm{GUT}})$ is reduced.
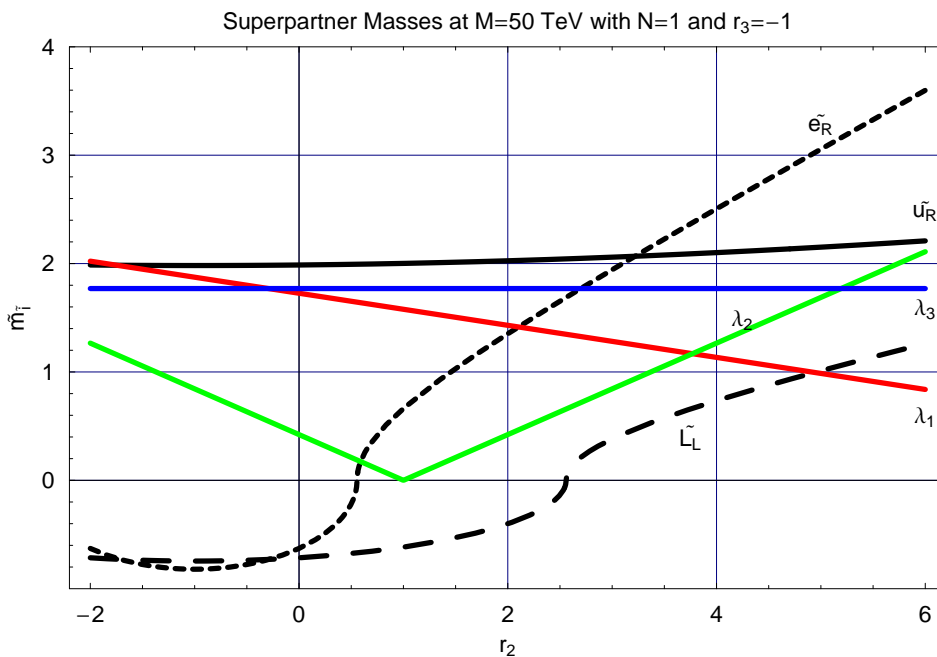
Figure 2: Spectrum of superpartner masses as a function of r_2 for $N = 1$, $M = 50$ TeV, and $r_3 = -1$. For gaugino masses we plot $|M|$ and for scalar masses, we plot $|m^2|^{1/2} \times \text{sgn}(m^2)$. All masses are in units of $F_\phi/(16\pi^2)$.
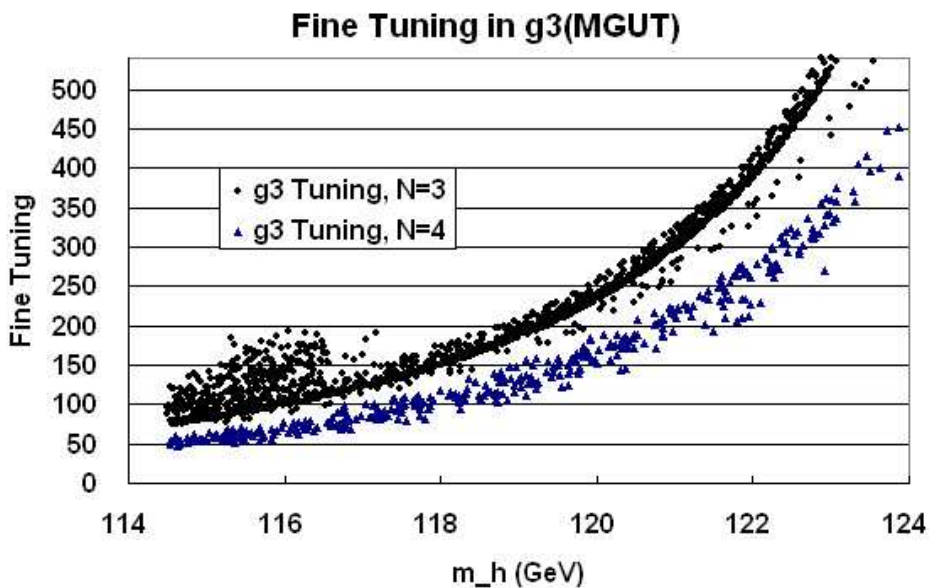

Figure 3: Fine-tuning in $g_3(M_{\text{GUT}})$ as a function of lightest Higgs mass m_{h^0} for models with $r > 0$ for $N = 3$ and 4.

5. Conclusion

We have constructed a well-motivated minimal model that naturally breaks SUSY in a

	Point 1	Point 2
N	1	4
r	14.6	6.45
F_ϕ	7.19 TeV	6.34 TeV
M	201 TeV	81 TeV
μ	485	425
$\tan\beta$	17.1	17.7
m_{h^0}	115	115
$m_{\tilde{l}L}$	380	330
$m_{\tilde{l}R}$	190	150
$m_{\tilde{q}L}$	1220	1170
$m_{\tilde{u}R}$	1170	1130
$m_{\tilde{d}R}$	1065	1120
$m_{\tilde{t}1}$	1070	1050
$m_{\tilde{t}2}$	1180	1150
$m_{\tilde{g}}$	880	1280
$m_{\tilde{\chi}_1^0}$	80	165
Tuning	170	55

Table 1: Sample MSSM spectra. All masses are in GeV. The main text gives the definition of fine-tuning.

flavor-blind way with a messenger scale near 10 TeV. The minimal model with one messenger has four continuous parameters and one discrete parameter, and can give rise to spectra that are very different from scenarios considered in the literature. These include "compact" spectra with colored superpartners close in mass to uncolored superpartners, a feature of the spectrum that may help with SUSY naturalness.

Acknowledgments

A. Minimization of the potential

We minimize eq. (2.1) assuming that all couplings are real. It is useful to write the potential as

$$V = \lambda_S^2 \left\{ \left| \tfrac{1}{2}S + \frac{c\langle F_\phi \rangle}{\lambda_S} \right|^2 |S|^2 + \left(\frac{c_S \langle F_\phi \rangle}{\lambda_S} \right)^2 \left(\frac{1}{2c_S}S^2 + \text{h.c.} \right) \right\} \tag{A.1}$$

and use units where $c_S \langle F_\phi \rangle / \lambda_S = 1$. We see that the phase structure is completely determined by the dimensionless parameter

$$\xi = \frac{1}{c_S}. \tag{A.2}$$

Writing

$$\langle S \rangle = s e^{i\theta}, \tag{A.3}$$

where s and θ are real, we have

$$\frac{V}{\lambda_S^2} = (1 + \xi \cos 2\theta)s^2 + s^3 \cos \theta + \tfrac{1}{4}s^4. \tag{A.4}$$

This is stationary in θ for

$$s = 0 \quad \text{or} \quad \sin \theta = 0 \quad \text{or} \quad \cos \theta = -\frac{s}{4\xi}. \tag{A.5}$$

We consider these cases one at a time.

The case $\sin \theta = 0$ is equivalent to $\langle S \rangle = s = \text{real}$. In that case, we find stationary points

$$s = s_\pm = \tfrac{1}{2}\left[-3 \pm \sqrt{1 - 8\xi}\right]. \tag{A.6}$$

Consistency therefore requires $\xi < \frac{1}{8}$. It is easy to check that

$$V(s_-) < V(s_+), V(0) \quad \text{for} \quad \xi < 0, \tag{A.7}$$

$$V(0) < V(s_-), V(s_+) \quad \text{for} \quad 0 < \xi < \tfrac{1}{8}. \tag{A.8}$$

It remains only to consider the third condition in eq. (A.5). In this case, the stationary points are

$$s = \tilde{s}_\pm = \pm 2\sqrt{\frac{\xi(\xi - 1)}{2\xi - 1}}. \tag{A.9}$$

Reality of s and $|\cos \theta| \leq 1$ are satisfied only if

$$\xi \geq 0. \tag{A.10}$$

We have $V(\tilde{s}_+) = V(\tilde{s}_-)$, as we expect since CP is spontaneously broken. We can check that

$$V(0) < V(\tilde{s}_\pm) \quad \text{for} \quad \xi < 1, \tag{A.11}$$

$$V(\tilde{s}_\pm) < V(0) \quad \text{for} \quad \xi > 1. \tag{A.12}$$

We conclude that

$$\langle S \rangle = \begin{cases} s_- & \xi < 0 \\ 0 & 0 < \xi < 1 \\ \tilde{s}_\pm e^{i\theta_\pm} & \xi > 1, \end{cases} \tag{A.13}$$

where

$$\cos \theta_\pm = \mp\sqrt{\frac{\xi - 1}{4\xi(2\xi - 1)}}. \tag{A.14}$$

Restoring the units, we obtain the formulas used in the main text.

B. Formulas for soft masses

In this appendix, we give some explicit one-loop formulas for SUSY breaking masses. The beta functions for the MSSM gauge couplings with N_2 doublets and N_3 triplets are

$$\beta_i = \frac{b_i}{16\pi^2} g_i^3, \tag{B.1}$$

where

$$b_3 = -3 + N_3, \tag{B.2}$$

$$b_2 = 1 + N_2, \tag{B.3}$$

$$b_1 = 11 + N_2 + \tfrac{2}{3} N_3. \tag{B.4}$$

The one-loop anomalous dimensions are

$$\gamma_{Q3} = \frac{1}{16\pi^2} \left[\frac{16}{3} g_3^2 + 3g_2^2 + \frac{1}{9} g_1^2 - 2y_t^2 \right], \tag{B.5}$$

$$\gamma_{u3} = \frac{1}{16\pi^2} \left[\frac{16}{3} g_3^2 + \frac{16}{9} g_1^2 - 4y_t^2 \right], \tag{B.6}$$

$$\gamma_{d3} = \frac{1}{16\pi^2} \left[\frac{16}{3} g_3^2 + \frac{4}{9} g_1^2 \right], \tag{B.7}$$

$$\gamma_{L} = \frac{1}{16\pi^2} \left[3g_2^2 + g_1^2 \right], \tag{B.8}$$

$$\gamma_{e} = \frac{1}{16\pi^2} \left[4g_1^2 \right], \tag{B.9}$$

$$\gamma_{Hu} = \frac{1}{16\pi^2} \left[3g_2^2 + g_1^2 - 6y_t^2 \right], \tag{B.10}$$

$$\gamma_{Hd} = \frac{1}{16\pi^2} \left[3g_2^2 + g_1^2 \right], \tag{B.11}$$

For the quark fields of the first and second generation, the top Yukawa coupling contribution should be dropped. We do not include the other Yukawa couplings, since they are negligible for small $\tan\beta$. The beta function for the top Yukawa coupling is

$$\beta_{yt} = \frac{y_t}{16\pi^2} \left[6y_t^2 - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{13}{9} g_1^2 \right]. \tag{B.12}$$

These formulas can be used to compute the MSSM soft masses using eqs. (2.19)–(2.22) in the main text. In the one-loop approximation, the contributions from the doublet and triplet messengers just add, and we obtain e.g.

$$m_{\tilde{Q},\text{AMSB}}^2 = \frac{1}{2} \frac{\langle F_\phi \rangle^2}{(16\pi^2)^2} \left[16g_3^4 - 3g_2^4 - \frac{11}{9} g_1^4 + 2y_t(16\pi^2 \beta_{yt}) \right], \tag{B.13}$$

$$\Delta m_{\tilde{Q}}^2 = \frac{1}{2} \frac{\langle F_\phi \rangle^2}{(16\pi^2)^2} \left[r_3(r_3+2) N_3 \left(\frac{16}{3} g_3^4 + \frac{2}{27} g_1^4 \right) \right.$$
$$\left. + r_2(r_2+2) N_2 \left(3g_2^4 + \frac{1}{9} g_1^4 \right) \right], \tag{B.14}$$

$$m_{\tilde{u}_R,\mathrm{AMSB}}^2 = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[16g_3^4 - \frac{176}{9}g_1^4 + 4y_t(16\pi^2\beta_{y_t})\right], \tag{B.15}$$

$$\Delta m_{\tilde{u}_R}^2 = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[\frac{16}{3}g_3^4 r_3(r_3+2)N_3\right.$$
$$\left.+\frac{16}{9}g_1^4\left(\frac{2}{3}r_3(r_3+2)N_3 + r_2(r_2+2)N_2\right)\right], \tag{B.16}$$

$$m_{\tilde{d}_R,\mathrm{AMSB}}^2 = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[16g_3^4 - \frac{44}{9}g_1^4\right], \tag{B.17}$$

$$\Delta m_{\tilde{d}_R}^2 = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[\frac{16}{3}g_3^4 r_3(r_3+2)N_3\right.$$
$$\left.+\frac{4}{9}g_1^4\left(\frac{2}{3}r_3(r_3+2)N_3 + r_2(r_2+2)N_2\right)\right], \tag{B.18}$$

$$m_{\tilde{L},\mathrm{AMSB}}^2 = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[-3g_2^4 - 11g_1^4\right], \tag{B.19}$$

$$\Delta m_{\tilde{L}}^2 = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[3g_2^4 r_2(r_2+2)N_2\right.$$
$$\left.+g_1^4\left(\frac{2}{3}r_3(r_3+2)N_3 + r_2(r_2+2)N_2\right)\right], \tag{B.20}$$

$$m_{\tilde{e}_R,\mathrm{AMSB}}^2 = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[-44g_1^4\right], \tag{B.21}$$

$$\Delta m_{\tilde{e}_R}^2 = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[4g_1^4\left(\frac{2}{3}r_3(r_3+2)N_3 + r_2(r_2+2)N_2\right)\right], \tag{B.22}$$

$$m_{H_u,\mathrm{AMSB}}^2 = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[-3g_2^4 - 11g_1^4 + 6y_t(16\pi^2\beta_{y_t})\right], \tag{B.23}$$

$$\Delta m_{H_u}^2 = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[3g_2^4 r_2(r_2+2)N_2 + g_1^4\left(\frac{2}{3}r_3(r_3+2)N_3 + r_2(r_2+2)N_2\right)\right], \tag{B.24}$$

$$m_{H_d,\mathrm{AMSB}}^2 = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[-3g_2^4 - 11g_1^4\right], \tag{B.25}$$

$$\Delta m_{H_d}^2 = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[3g_2^4 r_2(r_2+2)N_2 + g_1^4\left(\frac{2}{3}r_3(r_3+2)N_3 + r_2(r_2+2)N_2\right)\right]. \tag{B.26}$$

For the squarks of the first and second generation, we drop the top Yukawa coupling contribution.

The gaugino masses are given by

$$m_{\lambda_1} = \frac{\langle F_\phi\rangle}{16\pi^2}\left(-11 + \frac{2}{3}r_3 N_3 + r_2 N_2\right), \tag{B.27}$$

$$m_{\lambda_2} = \frac{\langle F_\phi\rangle}{16\pi^2}\left(-1 + r_2 N_2\right), \tag{B.28}$$

$$m_{\lambda_3} = \frac{\langle F_\phi\rangle}{16\pi^2}\left(3 + r_3 N_3\right), \tag{B.29}$$

where the first term in the parenthesis is the AMSB contribution while the remaining terms are contributions from the doublet and triplet messengers.

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